

# Trigonometric Identities & Formulas

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## Pythagorean Identities:

$$\cos^2\theta + \sin^2\theta = 1$$

$$1 + \tan^2\theta = \sec^2\theta$$

$$\cot^2\theta + 1 = \csc^2\theta$$

## Sum & Difference Identities:

$$\sin(\alpha \pm \beta) = \sin\alpha\cos\beta \pm \cos\alpha\sin\beta$$

$$\cos(\alpha \pm \beta) = \cos\alpha\cos\beta \mp \sin\alpha\sin\beta$$

$$\tan(\alpha \pm \beta) = \frac{\tan\alpha \pm \tan\beta}{1 \mp \tan\alpha\tan\beta}$$

## Double-Angle Identities:

$$\sin 2\theta = 2\cos\theta\sin\theta$$

$$\cos 2\theta = \cos^2\theta - \sin^2\theta$$

$$= 1 - 2\sin^2\theta$$

$$= 2\cos^2\theta - 1$$

$$\tan 2\theta = \frac{2\tan\theta}{1 - \tan^2\theta}$$

## Laws of Sines & Cosines:

$$\frac{\sin\alpha}{a} = \frac{\sin\beta}{b} = \frac{\sin\gamma}{c}$$

$$a^2 = b^2 + c^2 - 2bc\cos\alpha$$

$$b^2 = a^2 + c^2 - 2ac\cos\beta$$

$$c^2 = a^2 + b^2 - 2ab\cos\gamma$$

## Half-Angle Identities:

$$\sin^2\alpha = \frac{1 - \cos 2\alpha}{2} \Rightarrow \sin\frac{\beta}{2} = \pm \sqrt{\frac{1 - \cos\beta}{2}}$$

$$\cos^2\alpha = \frac{1 + \cos 2\alpha}{2} \Rightarrow \cos\frac{\beta}{2} = \pm \sqrt{\frac{1 + \cos\beta}{2}}$$

$$\tan^2\alpha = \frac{1 - \cos 2\alpha}{1 + \cos 2\alpha} \Rightarrow \tan\frac{\beta}{2} = \pm \sqrt{\frac{1 - \cos\beta}{1 + \cos\beta}} = \frac{1 - \cos\beta}{\sin\beta} = \frac{\sin\beta}{1 + \cos\beta}$$

## Product Formulas:

$$2\sin\alpha\sin\beta = \cos(\alpha - \beta) - \cos(\alpha + \beta)$$

$$2\cos\alpha\cos\beta = \cos(\alpha - \beta) + \cos(\alpha + \beta)$$

$$2\sin\alpha\cos\beta = \sin(\alpha + \beta) + \sin(\alpha - \beta)$$

$$2\cos\alpha\sin\beta = \sin(\alpha + \beta) - \sin(\alpha - \beta)$$

## Factoring Formulas:

$$\sin\alpha + \sin\beta = 2\sin\frac{\alpha + \beta}{2}\cos\frac{\alpha - \beta}{2}$$

$$\sin\alpha - \sin\beta = 2\cos\frac{\alpha + \beta}{2}\sin\frac{\alpha - \beta}{2}$$

$$\cos\alpha + \cos\beta = 2\cos\frac{\alpha + \beta}{2}\cos\frac{\alpha - \beta}{2}$$

$$\cos\alpha - \cos\beta = -2\sin\frac{\alpha + \beta}{2}\sin\frac{\alpha - \beta}{2}$$