Parametric and Polar Calculus Formulas

Calculus II ~ Prof. Sally J. Keely, M.S.

Calculus of Parametric Curves

Derivatives: $\dfrac{dy}{dx} = \dfrac{\frac{dy}{dt}}{\frac{dx}{dt}}$ $\qquad \dfrac{d^2y}{dx^2} = \dfrac{\frac{d}{dt}\left(\frac{dy}{dx}\right)}{\frac{dx}{dt}}$

Area between curve and x-axis: $\displaystyle\int_{t_1}^{t_2} y\left|\frac{dx}{dt}\right|dt$

Arc Length: $\displaystyle\int_{t_1}^{t_2}\sqrt{\left(\frac{dx}{dt}\right)^2+\left(\frac{dy}{dt}\right)^2}\,dt$

Surface Area

revolved about x-axis: $\displaystyle 2\pi\int_{t_1}^{t_2} y\sqrt{\left(\frac{dx}{dt}\right)^2+\left(\frac{dy}{dt}\right)^2}\,dt$

revolved about y-axis: $\displaystyle 2\pi\int_{t_1}^{t_2} x\sqrt{\left(\frac{dx}{dt}\right)^2+\left(\frac{dy}{dt}\right)^2}\,dt$

Calculus of Polar Curves

Derivative: $\dfrac{dy}{dx} = \dfrac{r\cos\theta + \frac{dr}{d\theta}\sin\theta}{-r\sin\theta + \frac{dr}{d\theta}\cos\theta}$

Distance between two polar pts: $\sqrt{r_1^{\,2}+r_2^{\,2}-2r_1r_2\cos(\theta_1-\theta_2)}$

Area inside polar curve: $\displaystyle\frac{1}{2}\int_{\alpha}^{\beta} r^2\,d\theta$

Arc Length: $\displaystyle\int_{\alpha}^{\beta}\sqrt{r^2+\left(\frac{dr}{d\theta}\right)^2}\,d\theta$

Surface Area

revolved about polar-axis: $\displaystyle 2\pi\int_{\alpha}^{\beta} r\sin\theta\sqrt{r^2+\left(\frac{dr}{d\theta}\right)^2}\,d\theta$

revolved about polar line $\theta=\frac{\pi}{2}$: $\displaystyle 2\pi\int_{\alpha}^{\beta} r\cos\theta\sqrt{r^2+\left(\frac{dr}{d\theta}\right)^2}\,d\theta$