Hyperbolic Trigonometric Identities & Formulas

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Definition Identities:

$$\sinh x = \frac{e^x - e^{-x}}{2} \qquad \tanh x = \frac{\sinh x}{\cosh x} \qquad \operatorname{csch} x = \frac{1}{\sinh x}$$

$$\cosh x = \frac{e^x + e^{-x}}{2} \qquad \coth x = \frac{\cosh x}{\sinh x} \qquad \operatorname{sech} x = \frac{1}{\cosh x}$$

Connection Identities:

$$\cosh^2 x - \sinh^2 x = 1$$

$$\cosh x + \sinh x = e^x \qquad 1 - \tanh^2 x = \operatorname{sech}^2 x$$

$$\cosh x - \sinh x = e^{-x} \qquad 1 - \coth^2 x = -\operatorname{csch}^2 x$$

Angle Identities:

$$\sinh 2x = 2\sinh x \cosh x$$

$$\cosh 2x = \cosh^2 x + \sinh^2 x = 2\sinh^2 x + 1 = 2\cosh^2 x - 1$$

$$\sinh^2 x = \tfrac{1}{2}(\cosh 2x - 1) \qquad \cosh^2 x = \tfrac{1}{2}(\cosh 2x + 1)$$

$$\sinh(x \pm y) = \sinh x \cosh y \pm \cosh x \sinh y$$

$$\cosh(x \pm y) = \cosh x \cosh y \pm \sinh x \sinh y$$

Inverse Hyperbolic Trig Function Formulas:

$$\operatorname{arcsinh} x = \ln(x + \sqrt{x^2 + 1}) \qquad \operatorname{arccosh} x = \ln(x + \sqrt{x^2 - 1}) \quad (x \geq 1)$$

$$\operatorname{arctanh} x = \tfrac{1}{2}\ln\frac{1+x}{1-x} \quad (|x| < 1) \qquad \operatorname{arccoth} x = \tfrac{1}{2}\ln\frac{x+1}{x-1} \quad (|x| > 1)$$

$$\operatorname{arcsech} x = \ln\left(\frac{1+\sqrt{1-x^2}}{x}\right) \quad (0 < x \leq 1) \qquad \operatorname{arccsch} x = \ln\left(\frac{1}{x} + \frac{\sqrt{x^2+1}}{|x|}\right) \quad (x \neq 0)$$